MICHELSON INTERFEROMETER & FOURIER TRANSFORM SPECTROSCOPY

WEEK 1: BUILDING AND UNDERSTANDING A MICHELSON INTERFEROMETER

LEARNING GOALS

1. Build and align a Michelson interferometer.
2. Construct a model of the simplest idealized model of the Michelson (50/50 transmission/reflection, same polarization, perfect alignment)
    a. Use the model to predict how moving one of the mirrors changes the power in each arm of the interferometer.
    b. Refine the model to account for non-ideal features.
3. Visually observe and explain how different kinds of misalignment generate different kinds of patterns between the two interfering beams. Use this information to optimize the interferometer alignment.
4. Apply the model and experimentally determined device parameters to estimate the smallest observable displacement change of one of the mirrors.

IDEAL INTERFEROMETER

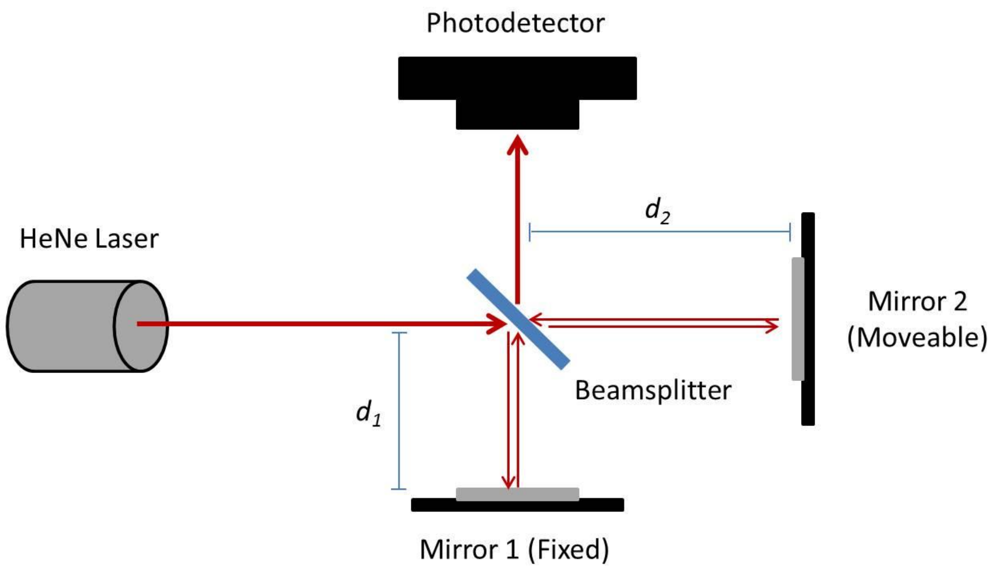


Figure 1: Michelson Interferometer

An ideal Michelson Interferometer will split the laser into two equally intense portions of the original beam with the beam splitter. The ideal model should also keep the beam perfectly collimated and have a uniform polarization.

Question 1	Assuming an idealized interferometer shown in Fig. 1 with a 50/50 beam splitter, single polarization of light, single wavelength, and perfect alignment, **predict the power incident on the photodetector P as a function of mirror position d_2** modeling the electric field of the light as plane waves of the form $\vec{E}_i(x,t) = E_i e^{i(kx-\omega t)}$, where $k = 2\pi/\lambda$ is called the wavenumber. • Write your prediction $P(d_2)$ as a mathematical expression and sketch the function also. • Explain how the four idealizations listed above are used in deriving $P(d_2)$. • How does the max and min power predicted by the function $P(d_2)$ compare to the power incident upon the beamsplitter?

BUILD AND ALIGN A MICHELSON INTERFEROMETER

Question 2	Set up a Michelson Interferometer using a HeNe laser as shown in Figure 1.
Question 3	Aligning a Michelson interferometer a. Carefully align the interferometer by an iterative process of overlapping the beams. This requires two tiltable mirrors prior to the beam splitter, one to overlap the beams at a point close to the beamsplitter, and the other to overlap the beams at a point far from the beam splitter. Why do you need 2 mirrors with 4 total knobs to get the beams overlapped? Why is one mirror not sufficient? (Question 4 will help you think through this in more detail.) b. Once the beam is in good alignment at both points, look in a far plane for visible interference fringes. The better the alignment the wider the fringe spacing will be. You can use this to further refine the alignment of your apparatus. c. How does the measured power vs position $P(x)$ compare to your prediction for the ideal case?
Question 4	Qualitatively predict and sketch what the visible interference pattern between the two recombined beams will look like for the following cases. For each case also sketch how the beam paths would differ from the ideal case to create this alignment condition and actually align your interferometer in this way to observe the pattern. a. Two Gaussian beams of equal size are perfectly overlapped, but incident at different angles. b. Two Gaussian beams of different size are perfectly overlapped and incident at the same angle. c. Two Gaussian beams of equal size and incident at the same angle, but are only partially overlapped. d. Briefly explain each of your sketches.

MEASURABLE PROPERTIES OF THE MICHELSON

The following steps will guide you through refining your idealized model to develop a more realistic model of the Michelson interferometer that you built.

Question 5	Without a perfect alignment, fringes will always be observable at large distances from the beam splitter. A small $\delta\Theta$ between the two incident beams can cause deviation from a good alignment. Sketch your prediction for how a small $\delta\Theta$ will affect $P(x)$ the power incident on the photodetector as a function of position.
Question 6	In the most simple model of the Michelson interferometer, we assumed power was distributed 50/50 from the beamsplitter. a. What is the actual power ratio between the two arms of your interferometer? What are the *electric field* reflection and transmission coefficients for the beam splitter? b. Use the coefficients to predict the power on the photodetector as a function of position $P(x)$. Compare your prediction to the ideal case and to measurements.
Question 7	Another assumption of the simple model was that the electric fields in each arm had identical polarization when the beams recombine. a. Should the beamsplitter in the Michelson be a polarizing beamsplitter or a non-polarizing beamsplitter? Why? b. Does the measured signal $P(x)$ depend on the incident polarization of the light? Is there a preferred polarization for the incident light on the beam splitter? c. If you rotate the polarization of one arm of the interferometer by $\pi/2$ before it recombines at the beamsplitter, how will the power on the photodetector $P(x)$ change from the idealized case? (If a quarter-wave plate, a.k.a $\lambda/4$ plate, is available, use it to change the polarization in one arm of the interferometer and test your answer.)
Question 8	One extremely common application of an interferometer is for sensitive measurement of displacement because very small displacements (sub-wavelength) cause large changes in the power measured on the photodetector. Use what you have learned about your particular Michelson interferometer to estimate the **position resolution of your interferometer**. This requires you estimate $\partial x/\partial P$ and determine an estimate for the smallest observable change in power δP_{min}. Explain all your reasoning.

WEEK 2: BUILDING A SPECTROMETER AND MEASURING FREQUENCY DATA

LEARNING GOALS

1. Incorporate a motorized translation stage into the existing interferometer.
2. Explain in simple terms how Fourier Transform Spectroscopy works.
3. Convert raw voltage vs. time data into a spectrum of intensity vs. frequency of light.
4. Explain how the parameters of the collected data affect the spectral resolution of the Fourier interferometer and how to choose the measurement parameters to achieve a desired resolution.

UNDERSTANDING FOURIER TRANSFORM SPECTROSCOPY

Spectroscopy in simplest terms is the analysis of the spectrum of a light source. Some forms of spectroscopy work by filtering all light except for a certain wavelength and measuring the intensity of the remaining light. In contrast, Fourier Transform Spectroscopy measures a broad spectrum of light at the same time. It works by creating constructive and destructive interference during the motion of one arm of an interferometer. When the moveable mirror is translated $\lambda/4$ the two interfering beams will go through a complete cycle of constructive and destructive interference. Each wavelength of light will have its own characteristic oscillation period as the mirror is translated. By taking a discrete Fourier transform of the Power vs Time data we can find the intensity of light vs. laser frequency.

Question 9	Write an expression to predict the photodetector voltage $V(t)$ as the mirror is being translated at a constant velocity, and assuming the light incident upon the interferometer is the combination of a 1 mW HeNe laser and a 0.5 mW 532 nm green laser.

The resulting waveform will be a function of intensity versus time or distance. To discern individual frequencies from one another a Discrete Fourier Transform must be applied to the wave to resolve the frequency, still as a function of time or distance.

Most data analysis programs have built in functions that can calculate the Fourier Transform for you. Once the data is collected we will take advantage of these. The following sections outline the experimental procedure, data collection, and analysis.

Question 10	a. In order to interpret the Fourier Transform of the raw data, you need to understand what optical frequency each element of the Fourier transform array corresponds to. Write an expression for the relation between the temporal frequencies of the recorded voltage of the interference signal and the actual optical frequency of the light. What is the laser frequency corresponding to the i-th element of the Fourier transform array? (The Lab Skill Activity on Fourier transforms may be helpful.) b. Test out your analysis by first generating a sample dataset of Voltage vs time using your expression for $V(t)$ in Question 9. Apply a discrete Fourier transform to this sample dataset to determine the frequency spectrum. You will use this same procedure later on with the experimental data. Your spectrum should show peaks at laser frequencies corresponding to the 633 nm and 532 nm wavelengths in the light.

CONVERTING THE INTERFEROMETER TO A SPECTROMETER

The following steps will help you convert your existing interferometer to a usable spectrometer to measure the frequency of any available light sources.

Question 11	a. Replace the micrometer dial on the translation stage of the interferometer with an electronic actuator. Then follow the setup instructions provided. See the instructions on setting up the motorized translation stages on the Hints page on the course website. b. Connect the photodetector (if you have not done so already) to the oscilloscope and the DAQ. Use the most up to date version of the VI you created in the lab skills activity on sampling and Fourier analysis. This VI should be able to accept the DAQ sample rate and number of samples as inputs. It should also save to file the time series of power data and plot the time series of voltage data as well as its Fourier transform for quick analysis of the data. c. Do a quick check to make sure the interferometer is well aligned. Use the week 1 alignment techniques to realign if necessary. Does moving the stage throughout its full range of motion change the alignment? Why?

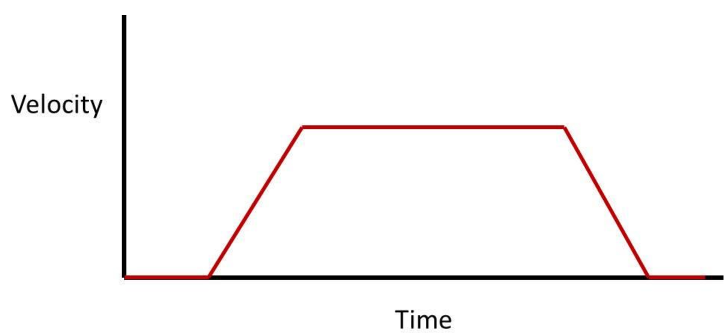
RECORDING DATA

A good dataset is crucial to the accuracy and resolution of the Fourier Transform for spectral analysis. Take a look back at the sampling rate LSA before collecting any data.

Question 12	a. According to the actuator specifications sheet, what are is range of motion, position resolution, and velocity resolution? b. What is the minimum sample rate you should choose for the DAQ assuming you have a HeNe laser and the translation stage velocity is v? c. *Qualitatively*, how does the distance the moveable arm travels affect resolution of the data? *Qualitatively*, how does sample rate affect the resolution of the data? How could this resolution be changed with the sample rate and number of samples parameters in your VI? d. *Quantitatively*, develop an expression that predicts frequency resolution of the ideal interferometer as a function of sample rate, number of samples, and velocity of the actuator. Use this expression to appropriately choose the number of samples and the sample rate for your spectral measurements.

Question 13	a. Set up the Thorlabs APT user control to make the actuator move a desired distance at a constant velocity. Make sure that the overall distance, and time to move that distance, exceed that of the sample you are recording. Why might you want to set up the experiment this way? b. Start by performing a few practice runs to ensure the spectrometer is functioning, the DAQ is recording data properly, and the parameters you chose are appropriate for your measurements. If anything is not working as expected, why? If it is a problem, resolve it. Now you are ready to take a few experimental data sets.

REFINING THE MODEL OF THE FOURIER TRANSFORM INTERFEROMETER

Question 14	a. What do you predict will happen to the voltage vs time data and the frequency spectrum if the translation stage does not move at a constant velocity during the scan? Sketch a graph of voltage vs. time assuming the velocity profile in Figure 2. How does this impact the resolution of the spectrometer?  **Figure 2: Velocity vs Time** b. How might you be able to minimize errors that result from inconsistent velocity? c. Experimentally determine if the velocity fluctuates. Do you have any evidence that it does change while it is supposed to be moving at a constant velocity?
Question 15	How does the photodiode responsivity $R(\lambda)$ affect the observed spectrum? How would you correct the Fourier transform spectrum to account for the different responsivity variation with wavelength?

PERFORMANCE SPECS OF THE SPECTROMETER

While the LabVIEW VI you are using includes a spectral analysis plot, it is often useful to perform the same analysis in a data analysis software package. This gives the user flexibility to perform further analysis or manipulation of the data.

Question 16	**Grand Challenge:** a. Gather experimental evidence and use it to determine the performance specs of your spectrometer. Use laser of known wavelength as a test source for the interferometer. In particular, what are the min and max measureable wavelengths and wavelength resolution? b. How does your observed resolution compare to the theoretical resolution of an idealized Fourier transform spectrometer? (See Question 12) c. If the observed resolution of the interferometer is worse than the theoretical resolution, what is the primary limiting factor? How would you redesign the spectrometer to improve the resolution?

FUTURE DIRECTIONS

a. **Phase modulation of a laser and measurement using homodyne detection**. Phase modulation is one way to encode information on a laser beam. Phase modulation can be achieved using a movable mirror or a liquid crystal phase modulator. Homodyne demodulation is a standard method for measuring a phase modulated signal.
b. **Interferometric measurement of the laser line width.** Measuring the intensity on the photodiode while changing the optical path lengths in the Interferometer gives a measurement of the autocorrelation function of the electric field. This autocorrelation can be used to determine the coherence time, the inverse of which is the line width.
c. **Measuring small displacements.** Any measurement that requires a very sensitive displacement measurement could use an interferometer as the displacement readout.